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A PHYSICALLY BASED DESCRIPTION FOR COUPLED PLASTICITY AND CREEP DEFORMATION

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Abstract—Based on a simple mechanical model and an appropriate definition of generalized time, a constitutive equation without using a yield surface is obtained for coupled plasticity and creep behavior of materials. The hardening of the materials is separated into two factors related respectively to inelastic strain range and nonproportionality. The steady creep of aluminum, copper and nickel under wide variation of equivalent stress, and the coupled plasticity and creep of 304 stainless steel subjected to non-steady state of biaxial stress at elevated temperature are analyzed. The validity of the proposed model is demonstrated by the satisfactory agreement between the experimental and calculated results. Since the proposed model does not use a yield surface, the corresponding numerical analysis for coupled plasticity and creep becomes greatly convenient. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

The coupled plasticity and creep behavior of the materials subjected to complex loading histories at elevated temperature has been intensively attractive in recent years due to the following motivations from which the new problems of load-bearing systems will mainly arise: to increase performance, to extend life to avoid failure (Leckie, 1985). For these purposes, materials will be required to work under more adverse operating conditions, so that, on one hand, high-performance materials are required and, on the other hand, a reliable estimation for operational life and failure analysis has to be performed. As its premise, the research on more realistic constitutive models becomes more and more significant.

Early in the 50s Besseling (1958) proposed a theory of plasticity and creep. From then on, remarkable progress has been made in the modeling of the constitutive behavior of materials at elevated temperature. With more and more understanding of the coupled plasticity and creep behavior, people no longer tend to simply separate the inelastic deformation into time-independent plastic and time-dependent creep parts, but treat it as a unified irreversible one caused by thermodynamic activation. Based on this concept various unified constitutive descriptions were proposed. Robinson (1978) proposed a constitutive relation by extending the potential theory of plasticity to include plasticity-creep interaction; Chaboche (1983), Krempl (1987), Phillips and Wu (1973) developed the descriptions for coupled plasticity-creep on the basis of nonlinear theory of viscoplasticity; Hart (1976), Miller (1976), Ponter and Leckie (1976) built up their models with various kind of microscopically based phenomenological constitutive theories. Freed and Walker (1993) proposed a viscoplastic theory, which reduces to creep and plasticity when time-dependent and time-independent conditions are considered, respectively. Based on the endochronic theory of viscoplasticity (Valanis, 1980), Watanabe and Atluri (1986) suggested a definition of intrinsic time, which leads to the evolution of the back stress that takes into account the thermal recovery as used in the famous Bailey–Orowan's relationship, and proved that Chaboche's model of viscoplasticity can be considered as one of its special cases. Wu and Chin (1995) investigated transient creep with a unified approach based on the endochronic constitutive equation. Murakami and Ohno (1982) proposed an elaborate creep model based on the notion of creep-hardening surface related to the dislocation motion under reversed stress history, which well described the creep of 304 stainless steel subjected to

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biaxial stress history. Meanwhile, systematically experimental studies on the coupled creep and plasticity of materials subjected to multiaxial complex loading histories have also been conducted and provided a series of significant results [see e.g. Murakami and Ohno (1986); Inoue *et al.* (1985)].

On the other hand, the constitutive models without using a yield surface were also developed and investigated [see e.g. Valanis (1980); Valanis and Fan (1983); Fan and Peng (1991); Murakami and Read (1987)], in which inelasticity is considered to be a gradually developing process which may be initially extremely small, but develops with increasing loading. These kind of models are also significant due to following statement by Drucker (1991): "the more sensitive the measurements that are made, the smaller will be the diameter of each yield surface. When a motion of a modest number of dislocations are detected as macroscopic plastic deformation the observed yield surface will shrink to zero size."

In this paper a unified constitutive relation without using a yield surface is proposed, based on a simple mechanical model and an appropriate definition of generalized time, for coupled plasticity and creep. The hardening of material is considered to consist of strain hardening and additional cross hardening. A hardening parameter ρ defined in inelastic strain space is introduced to take into account the effect of inelastic strain range; and the additional cross-hardening caused by nonproportional loading is considered by introducing a nonproportionality A. The proposed model is applied to the analysis of the steady creep of some typical metals under widely distributed stress levels and, the coupled creep and plasticity behavior of 304 stainless steel subjected to nonsteady biaxial complex loading. The obtained results are in satisfactory agreement with the experimental observation (Murakami and Ohno, 1986).

2. CONSTITUTIVE EQUATIONS

In the 60s Iwan (1966) considered a class of physically motivated models for the rate independent hysteretic behavior of materials. In the past few years, Fan and Peng (1991), Peng and Ponter (1994) proposed the constitutive equations for dissipative materials based on a simple mechanical model shown in Fig. 1, which can be extended to the analysis of coupled plasticity and creep. In Fig. 1, the deviatoric strain tensor $\mathbf{e}$ is separated into the elastic part $\mathbf{e}^e$ corresponding to the response of the spring E (with macroscopic elastic modulus G) and the inelastic part $\mathbf{e}^i$ corresponding to the response of the parallel branches. In the rth branch the dashpot-like block a_r (with "damping" coefficient a_r) and spring C_r (with stiffness C_r) are used to describe the rth dissipative mechanism. Noticing that during inelastic deformation, some part of energy may be stored in the residual stress fields on the microlevel due to the non-homogeneous nature of the stochastic internal structure of the material, and this part of energy is not dissipated and can be released under some condition. In this model, this part of energy is phenemonologically assumed to be stored in C_r

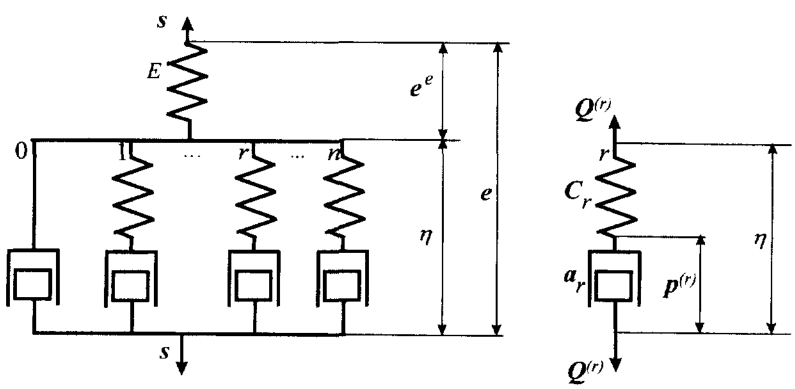

Fig. 1. A simple mechanical model for thermomechanically consistent constitutive relation.

($r = 1, 2, \ldots, n$). Assuming the material is initially isotropic and plastically incompressible, and in the case of isothermal and small deformation, one has the following relations

$$\mathbf{s} = \sum_{r=1}^{n} \mathbf{Q}^{(r)}, \tag{1}$$

$$\mathbf{Q}^{(r)} = C_r(T)(\mathbf{e}^{\mathrm{i}} - \mathbf{p}^{(r)}), \tag{2}$$

with

$$\mathbf{e}^{\mathrm{i}} = \mathbf{e} - \mathbf{e}^{\mathrm{e}} = \mathbf{e} - \frac{\mathbf{s}}{2G(T)}, \tag{3}$$

where $\mathbf{e}^{\mathrm{i}}$, $\mathbf{e}^{\mathrm{e}}$ and $\mathbf{e}$ represent the inelastic, elastic and total deviatoric strains, respectively, s the deviatoric stress, G shear modulus, T temperature, $\mathbf{p}^{(r)}$ and $\mathbf{Q}^{(r)}$ the rth deviatoric internal variable and the corresponding generalized force that satisfy the following dissipation inequality

$$\mathbf{Q}^{(r)} : \mathrm{d}\mathbf{p}^{(r)} \geqslant 0, \quad (r = 1, \ldots, n). \tag{4}$$

The generalized force $\mathbf{Q}^{(r)}$ is assumed to be related to the generalized irreversible flow rate, i.e. the derivative of the corresponding internal variable $\mathbf{p}^{(r)}$ with respect to z. In this work, the following simple linear relation is adopted

$$\mathbf{Q}^{(r)} = a_r(T)\frac{\mathrm{d}\mathbf{p}^{(r)}}{\mathrm{d}z}, \quad (r = 1, 2, \ldots, n), \tag{5}$$

in which z is generalized time. In coupled creep and plasticity the following relation (Valanis, 1980; Wu and Yip, 1980 is used

$$\mathrm{d}z^2 = \frac{k^2\,\mathrm{d}\zeta^2}{f^2} + \frac{\mathrm{d}t^2}{g^2}, \quad \mathrm{d}\zeta^2 = \mathrm{d}\mathbf{e}^{\mathrm{i}} : \mathrm{d}\mathbf{e}^{\mathrm{i}}, \tag{6}$$

where $f(T, \zeta)$ can be regarded as hardening function, $g(T, \zeta, \dot{\zeta})$ the function that takes into account the effect of inelastic deformation history on the viscous deformation, and $k(\dot{\zeta})$ strain rate sensitivity function. Wu and Yip (1980) suggested the following expression for $k(\dot{\zeta})$

$$k(\dot{\zeta}) = 1 - k_s \ln\left(\frac{\dot{\zeta}}{\dot{\zeta}_0}\right). \tag{7}$$

In which k_s is a material parameter and $\dot{\zeta}_0$ is the reference plastic strain rate. It is seen that $k(\dot{\zeta})$ decreases while $\dot{\zeta}$ increases and $k(\dot{\zeta}) = 1$ if $\dot{\zeta} = \dot{\zeta}_0$.

In the above relations C_r and a_r ($r = 1, \ldots, n$) should be positive due to the non-negative property of the stored energy and the dissipation [see eqn (4)]. The combination of eqns (1), (2) and (5) produces

$$\mathrm{d}\mathbf{Q}^{(r)} = C_r\,\mathrm{d}\mathbf{e}^{\mathrm{i}} + \left(-\alpha_r\,\mathrm{d}z + \frac{C_r'(T)}{C_r(T)}\mathrm{d}T\right)\mathbf{Q}^{(r)}, \quad \mathrm{d}\mathbf{s} = \sum_{r=1}^{n} \mathrm{d}\mathbf{Q}^{(r)} = C\,\mathrm{d}\mathbf{e}^{\mathrm{i}} + \mathrm{d}\mathbf{H}, \tag{8}$$

in which

$$C = \sum_{r=1}^{n} C_r, \quad \mathbf{dH} = \sum_{r=1}^{n} \left(-\alpha_r \, dz + \frac{C_r'(T)}{C_r(T)} dT \right) \mathbf{Q}^{(r)}, \tag{9}$$

$$\alpha_r = \frac{C_r(T)}{a_r(T)}, \quad C_r'(T) = \frac{dC_r(T)}{dT}. \tag{10}$$

This constitutive relation is similar to the endochronic constitutive relation without using a yield surface (Valanis, 1980), but the dependence of the material property on temperature is more emphasized. If $g^{-1} = 0$ and $k_s = 0$ this relation reduces to the description for time-independent plasticity, but in the sense that any real deformation takes time and should usually be time-dependent, we do not intend to define artificially a pure plastic process in the description for coupled plasticity and creep, but to extend the concept of plasticity to time-dependent process. During creep, $ds = 0$ so that the inelastic strain rate equals the total deviatoric strain rate; during stress relaxation, $d\mathbf{e} = 0$, so that the change of stress is related directly to the change of inelastic strain. For more exhaustive discussion, readers are referred to Wu and Chin (1995).

The incremental elastic response can be obtained from eqn (3) as

$$d\mathbf{s} = 2G(T)(d\mathbf{e} - d\mathbf{e}^i) + \frac{G'(T)}{G(T)} \mathbf{s} \, dT. \tag{11}$$

The volumetric response can be expressed by

$$\begin{aligned} \sigma_{kk} &= 3K(T)[\varepsilon_{kk} - 3\alpha(T)(T - T_0)] \\ d\sigma_{kk} &= 3K(T) \, d\varepsilon_{kk} + \left[\frac{K'(T)}{K(T)} \sigma_{kk} - 9\alpha'(T)K(T)(T - T_0) - 9\alpha(T)K(T) \right] dT, \end{aligned} \tag{12}$$

in which T_0 and T denote reference and current temperature, K and α elastic volumetric modulus and coefficient of thermal expansion, respectively.

As an example, for isothermal creep, $dT = d\mathbf{s} = 0$, one obtains the following relation by using eqns $(6)_2$ and (8)

$$C \, d\mathbf{e}^i = \mathbf{B} \, dz, \quad d\zeta^2 = \frac{\mathbf{B} : \mathbf{B}}{C^2} dz^2, \tag{13}$$

in which

$$\mathbf{B} = \sum_{r=1}^{n} \alpha_r \mathbf{Q}^{(r)}. \tag{14}$$

Combining eqns (6) and (13), one obtains

$$\frac{d\mathbf{e}^i}{dt} = \frac{f}{g} \left(\frac{\|\mathbf{B}\|}{fC} \right) \left[1 - \left(\frac{\|\mathbf{B}\|}{fC/k(\zeta)} \right)^2 \right]^{1/2} \frac{\mathbf{B}}{\|\mathbf{B}\|}. \tag{15}$$

It is seen that creep develops in the direction of $\mathbf{B}$ and the rate is determined by the applied stress and the internal structure of the material, which is phenomenologically characterized by $\mathbf{B}$, f and g. By assuming that g satisfies

$$\frac{1}{g}=\frac{k(\dot{\zeta})b(T)}{f}\left(\frac{\|\mathbf{B}\|}{fC/k(\dot{\zeta})}\right)^{m-1}\sqrt{1-\left[\frac{\|\mathbf{B}\|}{fC/k(\dot{\zeta})}\right]^2}\exp\left(n'\left\|\frac{s}{R_1}\right\|\right), \tag{16}$$

where $R_1 = \Sigma C_i/\alpha_i$, we have

$$\frac{d\mathbf{e}^i}{dt}=b(T)\left(\frac{\|\mathbf{B}\|}{fC/k(\dot{\zeta})}\right)^{m}\exp\left(n'\frac{\|s\|}{R_1}\right)\frac{\mathbf{B}}{\|\mathbf{B}\|}. \tag{17}$$

This relation is able to describe the transient process of creep when the current state of stress is changed to a new one. It is seen in eqns (1), (8), (14) and (17) that if s changes, $\mathbf{Q}^{(i)}$ $(i = 1, 2, \ldots, n)$, the components of $\mathbf{s}$ vary and approach the new values related to the new state of stress, which results in a change of $\mathbf{B}$ so that the creep rate varies transiently and approaches the corresponding steady-state accordingly.

3. HARDENING FUNCTION

The hardening/softening of the materials subjected to coupled plasticity and creep is macroscopically determined by the material properties, temperature and loading histories such as the nonproportionality and the range of inelastic strain path, etc. [see e.g. Murakami and Ohno (1986); Inoue *et al.* (1985); Freed and Walker (1993)]. Microscopically, it is related to the substructure of materials and thermal activation and recovery [see e.g. Murakami and Ohno (1982); Doong and Socie (1991); Bendersky *et al.* (1985)]. In our phenomenological description, the hardening function is, therefore, assumed to take the following multiplicatively separated form:

$$f = f_1 f_2, \tag{18}$$

where f_1 is the hardening factor corresponding to proportional loading, and f_2 is the additional hardening related to nonproportional loading history.

The evolution of f_i is assumed to take the following simple form [see e.g. Armstrong and Frederick (1966)]

$$df_i = \beta_i(d_i - f_i)\,dz \quad (i = 1, 2), \tag{19}$$

where d_i is the saturated value of f_i and β_i the rate for f_i to approach d_i. If d_i is constant eqn (19) reduces to the expression proposed by Wu and Yang (1983).

It is known that during inelastic deformation the saturated hardening is related to the inelastic strain range (Murakami and Ohno, 1982; Doong and Socie, 1991). In order to describe this effect, Peng *et al.* (1992) introduced a parameter ρ into the d_1 [see eqn (19)]:

$$d_1 = d_1(\rho), \tag{20}$$

in which ρ is determined by the following relations

$$g = (\mathbf{e}^i - \alpha)(\mathbf{e}^i - \alpha) - \rho^2 \leqslant 0, \tag{21}$$

where

$$\dot{\alpha} = (\tfrac{1}{2}\Gamma + \lambda\rho)\mathbf{u}^g\dot{\zeta} \tag{22}$$

$$\Gamma = \begin{cases} \mathbf{u}^g : \dot{\mathbf{e}}^i/\dot{\zeta} & \text{if } g = 0 \text{ and } \mathbf{u}^g : \dot{\mathbf{e}}^i \geqslant 0, \\ 0 & \text{otherwise} \end{cases}, \quad \mathbf{u}^g = \frac{\partial g/\partial \mathbf{e}^i}{\|\partial g/\partial \mathbf{e}^i\|}. \tag{23}$$

If λ [see eqn (22)] vanishes, the region defined by eqn (21) becomes identical to the non-hardening region proposed by Ohno (1982) for cyclic plasticity, or the creep-hardening region proposed by Murakami and Ohno (1982) for pure creep deformation.

Benallal and Marquis (1987) considered the effect of additional cross-hardening caused by nonproportional loading by introducing a measure of nonproportionality A into the parameters β_2 and d_2 in the hardening factor f_2, i.e.

$$\mathrm{d}_2(A) = \frac{pAd_{2\max}+(1-A)}{pA+(1-A)}, \quad \beta_2(A) = r_1 A + r_2(1-A) \tag{24}$$

where $d_{2\max}$ denotes the saturated value of f_2 when $A = 1$, p the material dependent weight parameter used to coordinate the material response corresponding to different non-proportionalities, and

$$A = 1-\cos^2\theta, \quad \cos\theta = \frac{\dot{\mathbf{e}}^{\mathrm{i}} : \dot{\mathbf{s}}}{\|\dot{\mathbf{e}}^{\mathrm{i}}\| \, \|\dot{\mathbf{s}}\|}. \tag{25}$$

Obviously, in the case of proportional loading $\dot{\mathbf{s}}$ is in the direction of $\dot{\mathbf{e}}^{\mathrm{i}}$ so that $A = 0$ (the minimum value). In the case of 90 out-of-phase loading which is regarded as the one with the largest nonproportionality, experiment for many materials showed that $A \to 1$ (the maximum value) [see e.g. Benallal and Marquis (1987)]. This definition of A can also be interpreted on the basis of dislocation motion: $\dot{\mathbf{e}}^{\mathrm{i}}$ and $\dot{\mathbf{s}}$ microscopically representing the variation of dislocation motion and the corresponding resistance, respectively, when non-proportional loading makes the maximum shear plane rotate, the hysteresis of the resistance will result in a phase difference between $\dot{\mathbf{e}}^{\mathrm{i}}$ and $\dot{\mathbf{s}}$.

4. APPLICATION AND VERIFICATION

4.1. *Material parameters and constants*

The material parameters and constants involved in this model are listed as follows:

(1) G, E are the shear and Young's moduli, respectively, which may be temperature-dependent.

(2) C_i, α_i are the parameters of inelastically, in which C_i may be temperature dependent, α_i is the ratio between C_i and a_i [see eqn (10)]. For simplicity, it is assumed that both C_i and a_i vary with respect to temperature by the same rule, so that α_i becomes independent of temperature.

(3) d_1, β_1, λ are the saturated hardening parameter, hardening rate parameter and material dependent constant related to the fading memory of hardening state, respectively. d_1 may be related to the hardening parameter ρ which, in turn, is determined by eqns (21)–(23).

(4) r_1, r_2 are the rates of additional cross-hardening and the hardening relaxation, respectively.

(5) $d_{2\max}$, p are the maximum additional cross-hardening that occurs at $A = 1$, and parameter to coordinate nonproportional hardening results from various inelastic strain paths.

(6) k_s is the strain rate sensitive parameter.

(7) m, n', b_0 are the constants for the description of creep deformation.

These material parameters and constants can be identified by the following two steps: first, plasticity processes with a constant strain rate are considered, so that the parameters and constants in the items (1)–(5) and their temperature dependence (if different temperatures are chosen for the tests) can be determined in the way discussed by Fan and Peng (1991). In the second step, changing plastic strain rate and considering pure creep or relaxation processes the constants and parameters in items (6) and (7) can be determined.

4.2. *Analysis for steady creep of aluminum, copper and nickel*

In this part the steady creep states of some typical metals under widely scattered stress levels will be analyzed by using the proposed model. In order to simplify the discussion, $n = 1$ [see eqn (1)], $f = 1$ and $k = 1$ are chosen in this part without losing generality. Letting [see e.g. Freed and Walker (1993)]:

$$b(T) = b_0\theta(T) \tag{26}$$

where

$$\theta(T) = \begin{cases} \exp\left(-\dfrac{Q_0}{k_0}T\right) & T_t \leqslant T < T_m \\ \exp\left[-\dfrac{Q_0}{k_0}T_t\left(\ln\left(\dfrac{T_t}{T}\right)+1\right)\right] & 0 < T \leqslant T_t \end{cases} \tag{27}$$

T_t and T_m are transition temperature and melting temperature, respectively, the following relation is obtained immediately from eqn (17)

$$\|\dot{\mathbf{e}}^i\| = \theta(T)Z_{ss}\left(\frac{\|\mathbf{s}\|}{R_1}\right) \tag{28}$$

in which $R_1 = C_1/\alpha_1$ and

$$Z_{ss}\left(\frac{\|\mathbf{s}\|}{R_1}\right) = b_0 \cdot \left(\frac{\|\mathbf{s}\|}{R_1}\right)^m \exp\left(n'\frac{\|\mathbf{s}\|}{R_1}\right) \tag{29}$$

is the expression of the steady-state Zener parameter from the proposed model.

The variation of the Zener parameters vs equivalent stress for aluminum, copper and nickel calculated by using eqn (29) is shown in Fig. 2(a)–(c), respectively. The constants involved are determined and listed in Table 1, in which the constants Q_0 and T_m are directly quoted from Freed and Walker (1993), but are not used in the calculation of z_{ss}, the parameter R_1 is obtained by the material constants related to the material strength, and b_0, m and n' are determined by the corresponding experimental data (Freed and Walker, 1993). It is seen that the proposed model can satisfactorily describe the experimental steady-state creep rates (all the experimental results are indirectly quoted from Freed and Walker, 1993), in a wide range of stress levels and for different metals.

4.3. *Application to coupled creep and plasticity of* 304 *stainless steel subjected to non-steady biaxial states of stress*

The coupled plasticity and creep of 304 stainless steel subjected to non-steady biaxial states of stress at 600°C is to be analyzed. By choosing $n = 3$ [see e.g. Valanis and Fan (1983)] and

$$d_1(\rho) = 1+\gamma_1\rho^{\gamma_2} \tag{30}$$

for simplification, and noticing the deformation process is isothermal, the material constants are determined as

$$G, E = 54{,}135\,\text{GPa} \quad C_{1,2,3} = 8.2\times10^5, 2.4\times10^4, 2.5\times10^3\,\text{MPa}$$

$$\alpha_{1,2,3} = 1.37\times10^4, 3.80\times10^2, 4.2\times10^1 \quad \gamma_1, \gamma_2, \beta_1, \lambda = 1050, 1, 8, 5$$

$$r_1, r_2, d_{2\max}, p = 4000, 6, 2.5, 6 \quad b_0, m, n' = 2.8\times10^{-6}, 18, 121$$

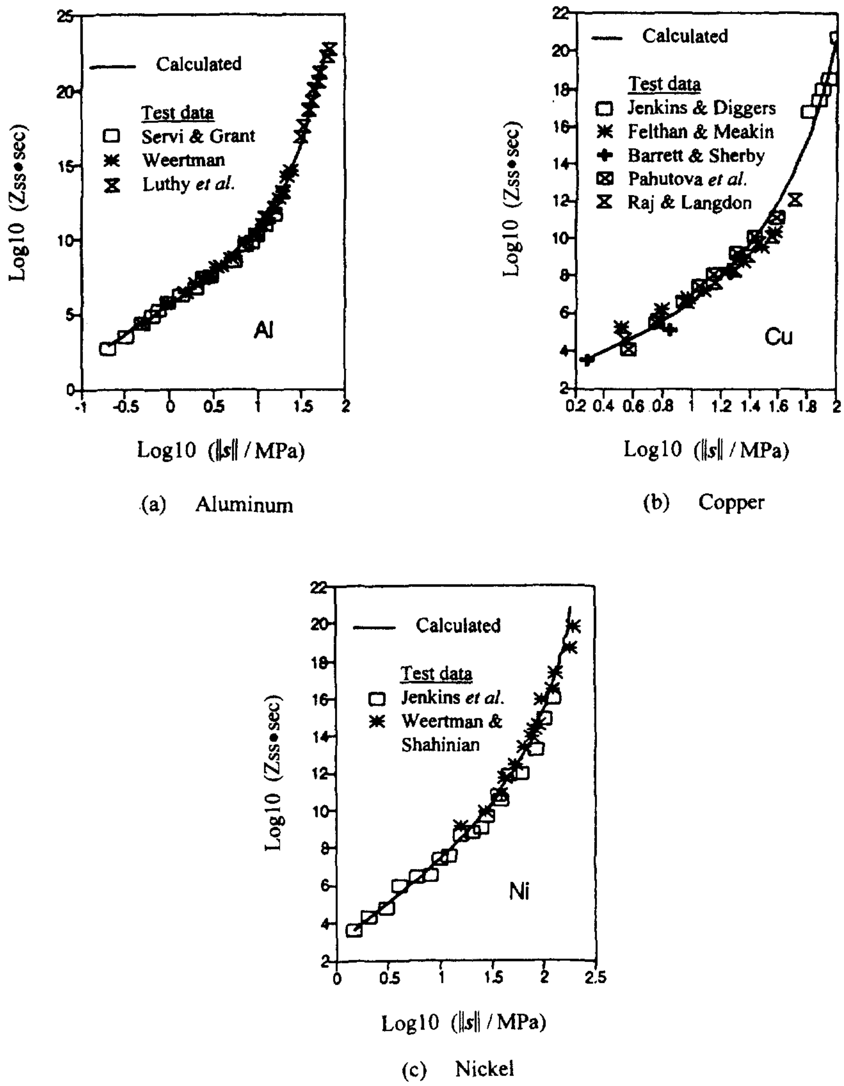

Fig. 2. Stationary creep behavior: (a) aluminum; (b) copper; (c) nickel.

Table 1. Steady-state creep constants

Constants	Al	Cu	Ni
b_0 (s^{-1})	3.86×10^{12}	5.93×10^{8}	8.94×10^{12}
R_1 (MPa)	84.92	138	265.4
m	3.79	3.97	4.19
n'	34.39	38.36	24.95
Q_0 (J/mol)	1.4×10^{5}	2.0×10^{5}	2.9×10^{5}
T_m (K)	933	1356	1726

$$k_s = 0.006 \text{ (given } \dot{\zeta}_0 = 0.01\ s^{-1}\text{)}.$$

In the biaxial stress and strain spaces the stress and strain vectors are defined as

$$\boldsymbol{\sigma} = \sigma\mathbf{n}_1 + \sqrt{3}\tau\mathbf{n}_2, \quad \boldsymbol{\varepsilon} = \varepsilon\mathbf{n}_1 + \frac{\gamma}{\sqrt{3}}\mathbf{n}_2, \tag{31}$$

where σ, τ stand for normal- and shear-stress components, ε, γ normal- and shear-strain components, respectively. $\mathbf{n}_1$ and $\mathbf{n}_2$ are a set of normalized unit vectors. The equivalent stress and strain can then be expressed as

$$\bar{\sigma} = |\boldsymbol{\sigma}| = \sqrt{\sigma^2 + 3\tau^2}, \quad \bar{\varepsilon} = |\boldsymbol{\varepsilon}| = \sqrt{\varepsilon^2 + \frac{1}{3}\gamma^2}. \tag{32}$$

The coupled creep and plasticity is analyzed by the incremental approach. For readers to assemble the proposed model for code implementation and application more easily, the numerical approach for the isothermal case, i.e. $\Delta T = 0$ (but this approach can also be applied to the case of $\Delta T \neq 0$ without difficulty), is briefly introduced as follows:

Suppose the analysis for the nth increment of loading has been finished and one obtains $\mathbf{s}_n$, t_n, $\mathbf{Q}_n^{(r)}$ $(r = 1, 2, 3)$, $\mathbf{B}_n$, z_n, $\mathbf{e}_n^{\mathrm{i}}$, ζ_n, $\mathbf{e}_n$, ρ_n, A_n, f_n and $k_n(\dot{\zeta})$. Given $(n+1)$th increment of stress Δs_{n+1} and the corresponding increment of time Δt_{n+1}, one can have Δz_{n+1} immediately by solving the following equation obtained by substituting eqns (6) and (9) into eqn (8) and using eqn (14) [see Murakami and Read (1987)]

$$\left(\frac{f_n^2}{k_n^2} C^2 - \mathbf{B}_n : \mathbf{B}_n\right) \Delta z_{n+1}^2 + 2\mathbf{B}_n : \Delta \mathbf{s}_{n+1} \Delta z_{n+1} - \left(\Delta \mathbf{s}_{n+1} : \Delta \mathbf{s}_{n+1} + \frac{f_n^2 C^2}{g_n^2 k_n^2} \Delta t_{n+1}^2\right) = 0 \tag{33}$$

then obtain $\Delta \mathbf{e}_{n+1}^{\mathrm{i}}$ from eqns (8) and (9), $\Delta \zeta_{n+1}$ from eqn $(6)_2$, $\Delta \mathbf{Q}_{n+1}^{(r)}$ $(r = 1, 2, 3)$ from eqn $(8)_1$.

$$\mathbf{e}_{n+1}^{\mathrm{i}} = \mathbf{e}_n^{\mathrm{i}} + \Delta \mathbf{e}_{n+1}^{\mathrm{i}} \tag{34}$$

$\Delta \rho_{n+1}$ can be determined with eqns (22), (23) and the condition of consistence $\dot{g} = 0$ [see eqn (21)], and then d_1 and f_1 can be calculated by eqns (20) and (19), respectively. Meanwhile, making use of the given $\Delta \mathbf{s}_{n+1}$ and the derived $\Delta \mathbf{e}_{n+1}^{\mathrm{i}}$, A, d_2, β_2 and f_2 can be calculated by eqns (25), (24) and (19) sequentially, and the hardening function f can be obtained with eqn (18). Then, by superimposing the increments onto the corresponding results after nth increment of loading, one obtains $\mathbf{s}_{n+1}$, t_{n+1}, $\mathbf{Q}_{n+1}^{(r)}$ $(r = 1, 2, 3)$, $\mathbf{B}_{n+1}$, A_{n+1}, f_{n+1} and $k_{n+1}(\dot{\zeta})$ and then starts the next incremental loading. This unified approach is used in all the following analyses. It is seen that the numerical process based on the proposed model for the coupled plasticity and creep analysis is quite simple and direct.

Figures 3–9 show the responses of 304 stainless steel subjected to various loading histories. The experimental results in Figs 3, 4 and 7 were used to determine the material constants. In Fig. 3 the experimental equivalent stress and plastic strain relation (Murakami

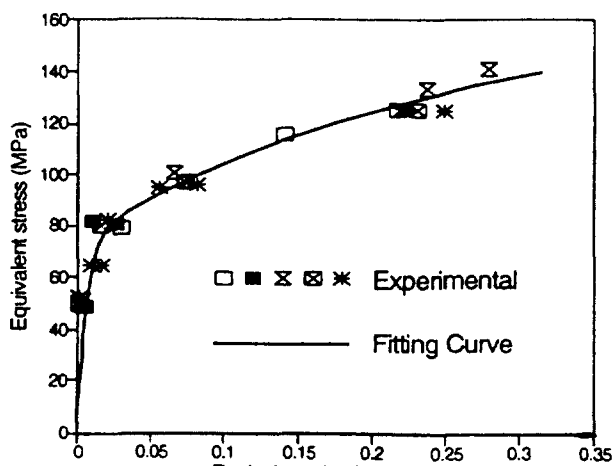

Fig. 3. Equivalent stress vs equivalent strain obtained at the incipient loading in creep test.

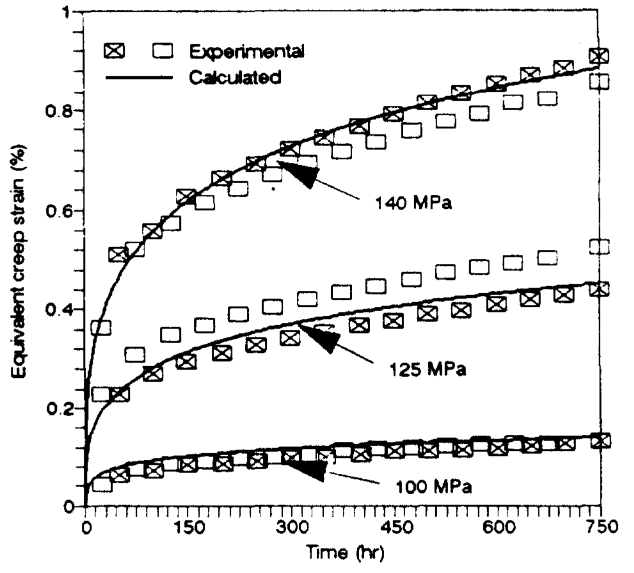

Fig. 4. Constant tension and torsion curves and the succeeding creep recovery curves.

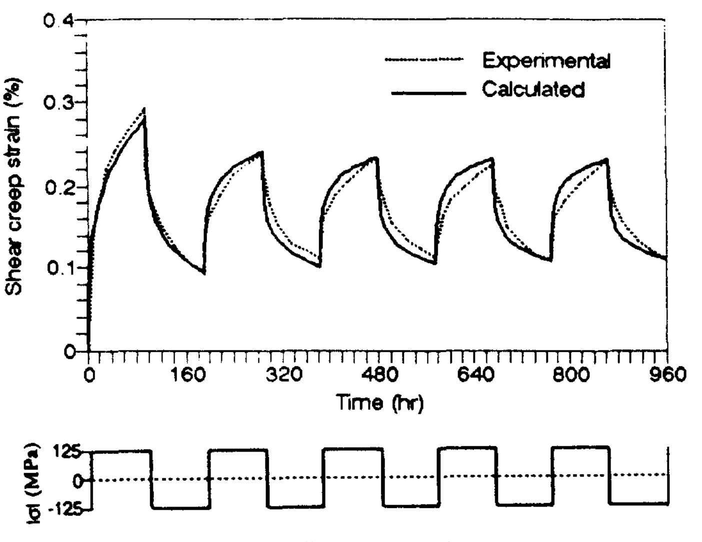

Fig. 5. Cyclically reversed torsion test.

et al., 1986) is shown with symbols and the fitting curve with solid line. The loading stress process is stress-controlled with equivalent-stress rate of 7 MPa/s (in the following the same rate is also used in the calculation when the state of stress changes).

The symbols in Fig. 4 show the experimental relation between the equivalent creep strain and time (Murakami *et al.*, 1986). It is seen that there is a long transient process before steady-creep state is reached, both the accumulated creep strain and the creep rate at steady-state greatly increases with the increase of applied stress. The solid lines are the fitting curves, which well describe both the transition and the steady-state of the creep under different stress level.

Figure 5 shows the variation of creep strain under cyclic reversed torsion. The dashed curve corresponds to the experimental result (Murakami *et al.*, 1986). It is seen that the amplitude of creep strain decreases in the first few cycles and then almost remains constant, which may be attributed to material hardening. The solid curve denotes the calculated result, which is in good agreement with the experiment.

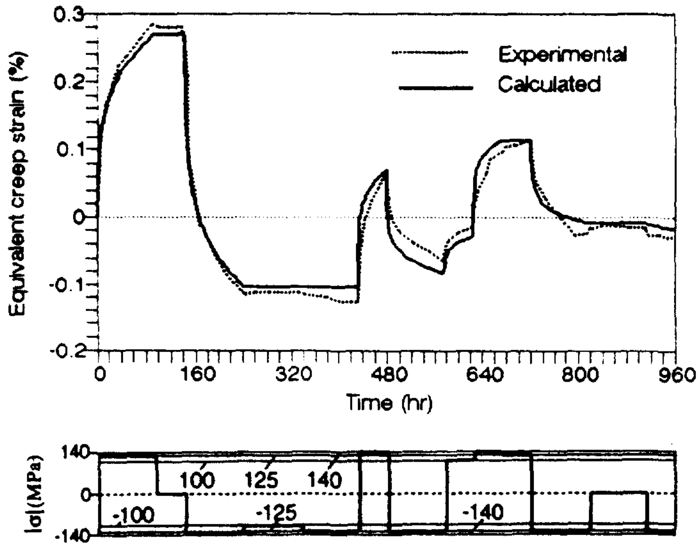

Fig. 6. Torsional creep test under random variation of stress magnitude.

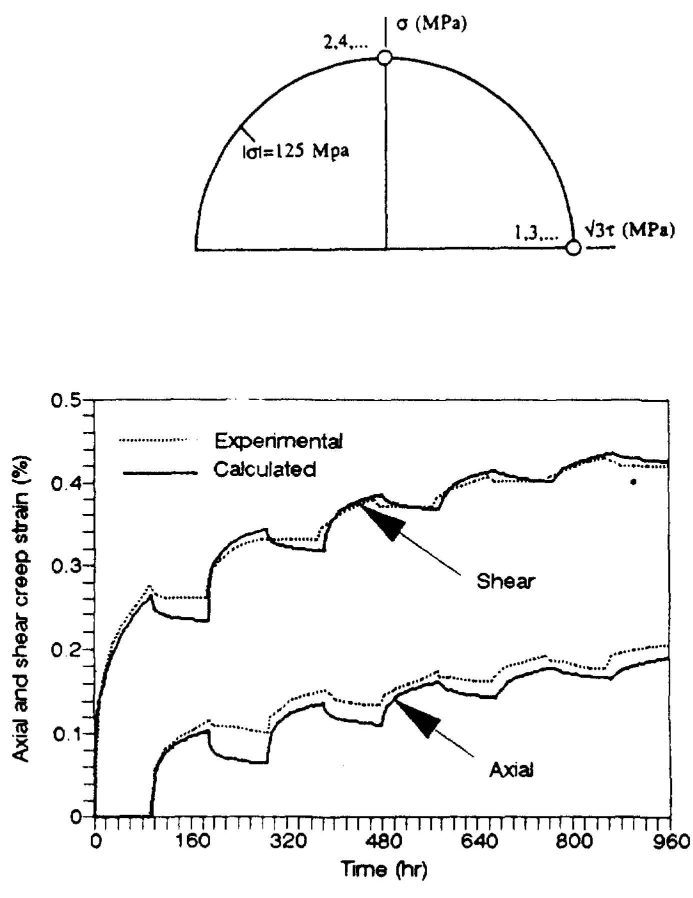

Fig. 7. Combined tension and torsion test.

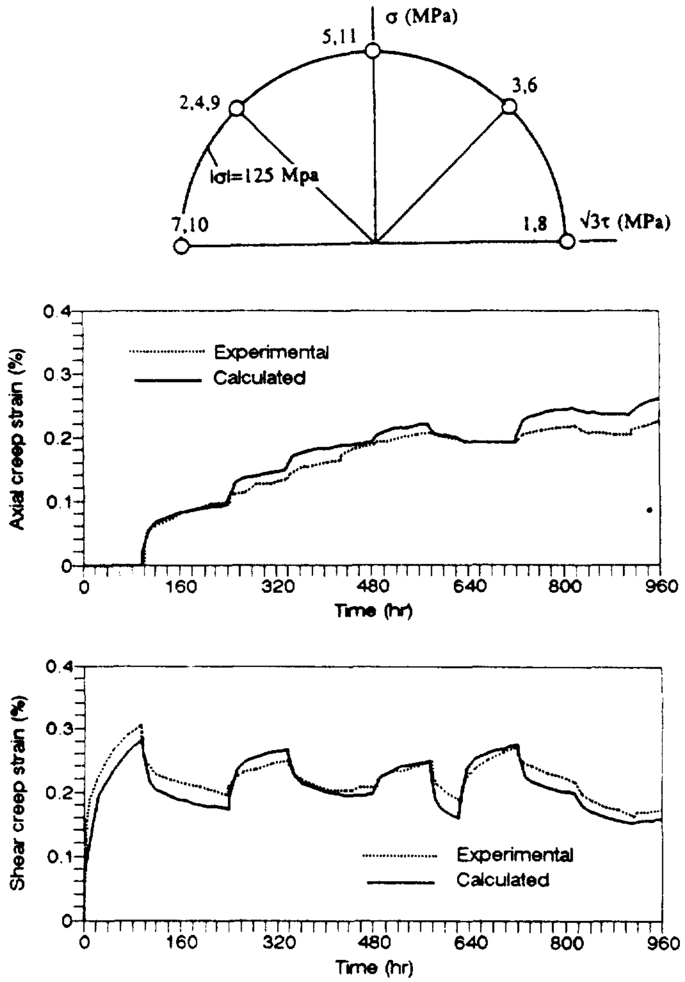

Fig. 8. Creep test under constant equivalent stress and random variation in principal stress direction and interval of stress change.

The relation between creep strain and time under the stress history with irregularly varying magnitude and interval is shown in Fig. 6. It is observed that the increase of creep rate is more significant when the sign of stress is reversed (Murakami *et al.*, 1986). The proposed model is able to describe the main features in this deformation process.

Figures 7–9 correspond to the responses of the material subjected to biaxial cyclic plasticity and reversed creep. The stress path is shown in the upper part of each figure. Each loading history consists of a loading from the origin to point 1 where the stress was kept constant for Δt_1 hours followed by a complete unloading, and then a subsequent loading to point 2 where the stress was kept constant for Δt_2 hours followed by a complete unloading again, ... The time interval at each point in the corresponding stress diagram are shown in Table 2.

Additional cross-hardening is observed (see Fig. 7) when the loading direction is changed. The experimental result in this figure was used for the determination of the material constants involved in additional cross-hardening. The correlation shows that the proposed model can describe the behavior under biaxial cyclic plasticity and reversed creep. The transient increase of creep strain and the succeeding creep recovery after the alternation of tensile and shear stress can also be well described.

The responses of the material under random variation of multiaxial states of stress are shown in Figs 8 and 9. In Fig. 8 the direction and the interval of stress change irregularly, but the equivalent stress magnitudes keep constant. It is found that the increase of creep

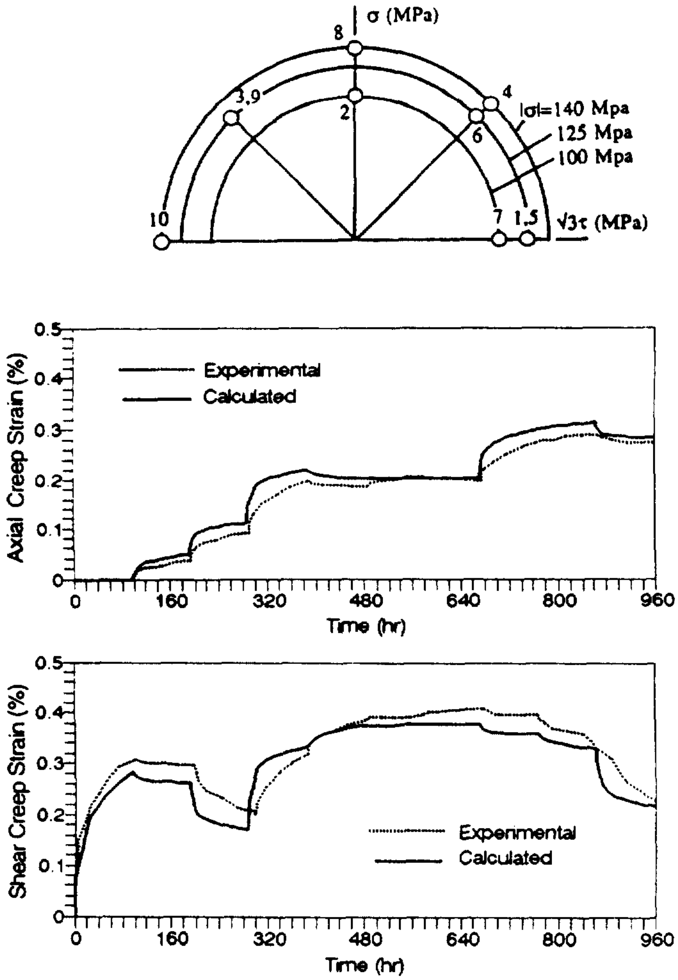

Fig. 9. Creep test under random variation in equivalent stress and principal stress direction, but constant interval of stress change.

Table 2. Time interval for creep (hr)

Point	1	2	3	4	5	6	7	8	9	10	11
Fig. 7	96	96	96	96	96	96	96	96	96	96	—
Fig. 8	96	144	96	96	48	96	48	96	96	96	48
Fig. 9	96	96	96	96	96	96	96	96	96	96	—

rate due to the change of stress state is more significant after larger variation of stress direction and this tendency is satisfactorily described by the proposed model.

The result in Fig. 9 corresponds to the stress history in which both the magnitude and the direction of stress varies randomly, but the interval of stress variation remains constant. It is seen that creep develops faster when either the variation of stress direction is larger or the amplitude of stress increases. The comparison between experimental and calculated results shows reasonable agreement.

5. CONCLUSIONS

A unified constitutive description for coupled plasticity and creep is derived from a simple mechanical model and by introducing an appropriate definition of generalized time.

The hardening function is multiplicatively separated into two hardening factors f_1 and f_2, corresponding, respectively, to cyclic hardening and additional cross hardening.

The steady-state creep rate of aluminum, copper and nickel subjected to widely distributed equivalent stresses were analyzed and the calculated result well correlates the experimental observation. The proposed model was also applied to the description of the plastic and creep behavior of 304 stainless steel subjected to nonsteady biaxial states of stress, and the validity is demonstrated by the comparison between the calculated and the experimental results (Murakami *et al.*, 1986).

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